The axial-vector form factor in QCD

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Lattice meets experiment

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Based on work with
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Outline

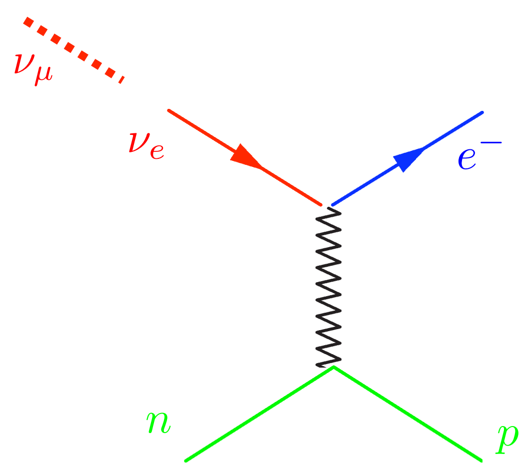
- nuclear physics and philosophy
- math
- particle (nucleon) physics
- summary

Nuclear physics and philosophy

neutrino-nucleus scattering involves three hard problems

σ ~ flux x |nucleon amplitude|^2 x nuclear effects

~measurement ~model ~model



Degenerate uncertainties. E.g., charged current quasi-elastic scattering (CCQE) used as flux monitor, to determine nucleon axial-vector form factor, and to constrain nuclear modeling *would like to see this as*

$$\sigma \sim \text{flux} \times |\text{nucleon amplitude}|^2 \times \text{nuclear effects}$$

~measurement ~lattice ~direct experimental constraints

- some progress and many proposals experimentally on constraining nuclear models

- relevant accuracy of nucleon amplitudes within range of lattice simulations

these problems are hard, but important

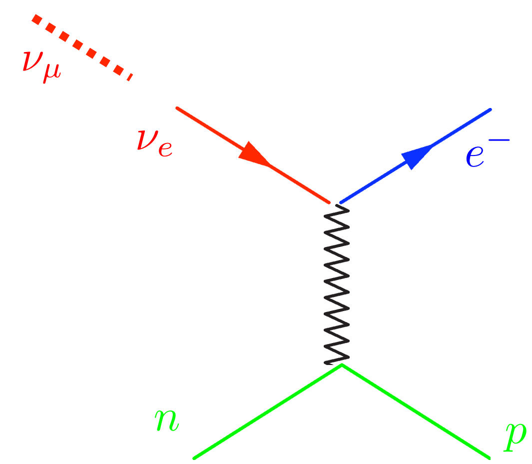
scattering of ~GeV leptons on a nucleus far from optimal theoretically

- above nuclear, chiral perturbation theory scales
- below scale of QCD perturbation theory, inclusive observables

Driven to this regime by several considerations

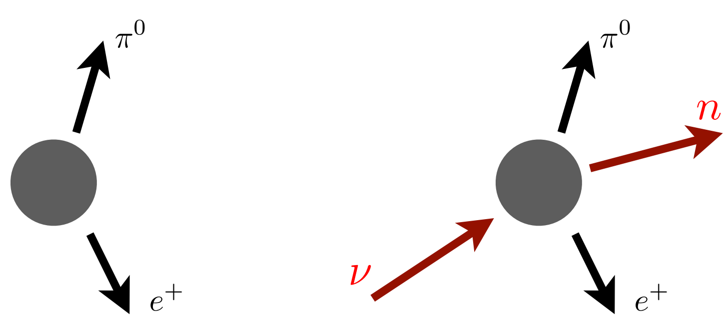
- neutrino oscillations for hierarchy and CP violation

$$P(\nu \to \nu') = \sin^2 2\theta \sin^2 \left[1.27 \, \Delta m^2 (\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})} \right]$$



- proton decay and related atmospheric backgrounds

$$m_p \sim \mathrm{GeV}$$



cracks in the foundation

the problems of the nucleon-level amplitude and nuclear modeling have been dominated by default ansatze:

Dipole ansatz

[e.g., Llewellyn-Smith, Phys.Rept. 3 (1972) 261-379]

- analyticity + lattice QCD: model independent determination (focus of this talk)

Relativistic Fermi Gas (RFG) ansatz

[R.A. Smith and E. J. Moniz, Nucl. Phys. B43, 605(1972), B101, 547(E) (1975)]

- multiple experimental programs proposed to constrain the hadronic final state, e.g., 3 at most recent Fermilab PAC
- significant nuclear modeling, MC generator efforts to improve upon RFG

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The actors:

Constrained by electron scattering

$$\bar{u}^{(p)}(p')\Gamma^\mu u^{(n)}(p) = \langle p(p')|J_W^{+\mu}|n(p)\rangle$$

$$\propto \bar{u}^{(p)}(p')\Big\{\gamma^\mu F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2(q^2)$$

$$+ \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\Big\}u^{(n)}(p)$$

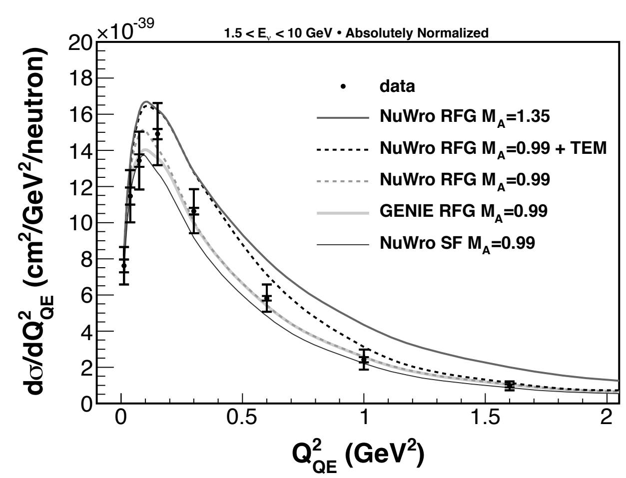
Axial form factor

suppressed by lepton mass, and constrained by PCAC

Dipole ansatz *[e.g., Llewellyn-Smith, Phys.Rept. 3 (1972) 261-379]*

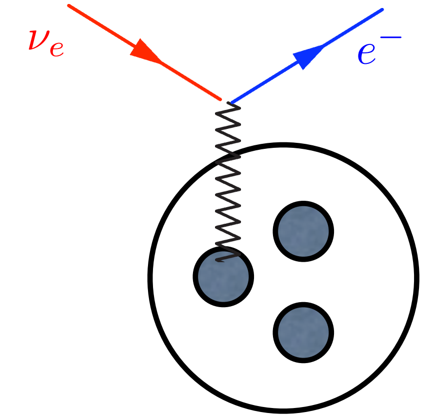
$$F_A(q^2) = \frac{1}{\pi}\int_{t_{\rm cut}}^\infty dt \frac{{\rm Im}F_A(t+i0)}{t-q^2} \to \frac{g_A}{\left(1-\frac{q^2}{m_A^2}\right)^2}$$

Agrees with asymptotic ~1/Q^4 behavior, but physically relevant region is far from asymptotic



[MINERvA, Phys.Rev.Lett. 111 (2013) 022502]

Nuclear model



$$\sigma_{\text{nuclear}} = \frac{G_F^2}{16|k \cdot p_T|} \int \frac{d^3k'}{(2\pi)^3 2E_{\boldsymbol{k}'}} L^{\mu\nu} W_{\mu\nu},$$

Relativistic Fermi gas

[R.A. Smith and E. J. Moniz, Nucl. Phys. B43, 605(1972), B101, 547(E) (1975)]

$p_F,\ A$

$$W_{\mu\nu} \equiv \int d^3p\, f(\boldsymbol{p}, q^0, \boldsymbol{q}) H_{\mu\nu}(\epsilon_{\boldsymbol{p}}, \boldsymbol{p}; q^0, \boldsymbol{q}),$$

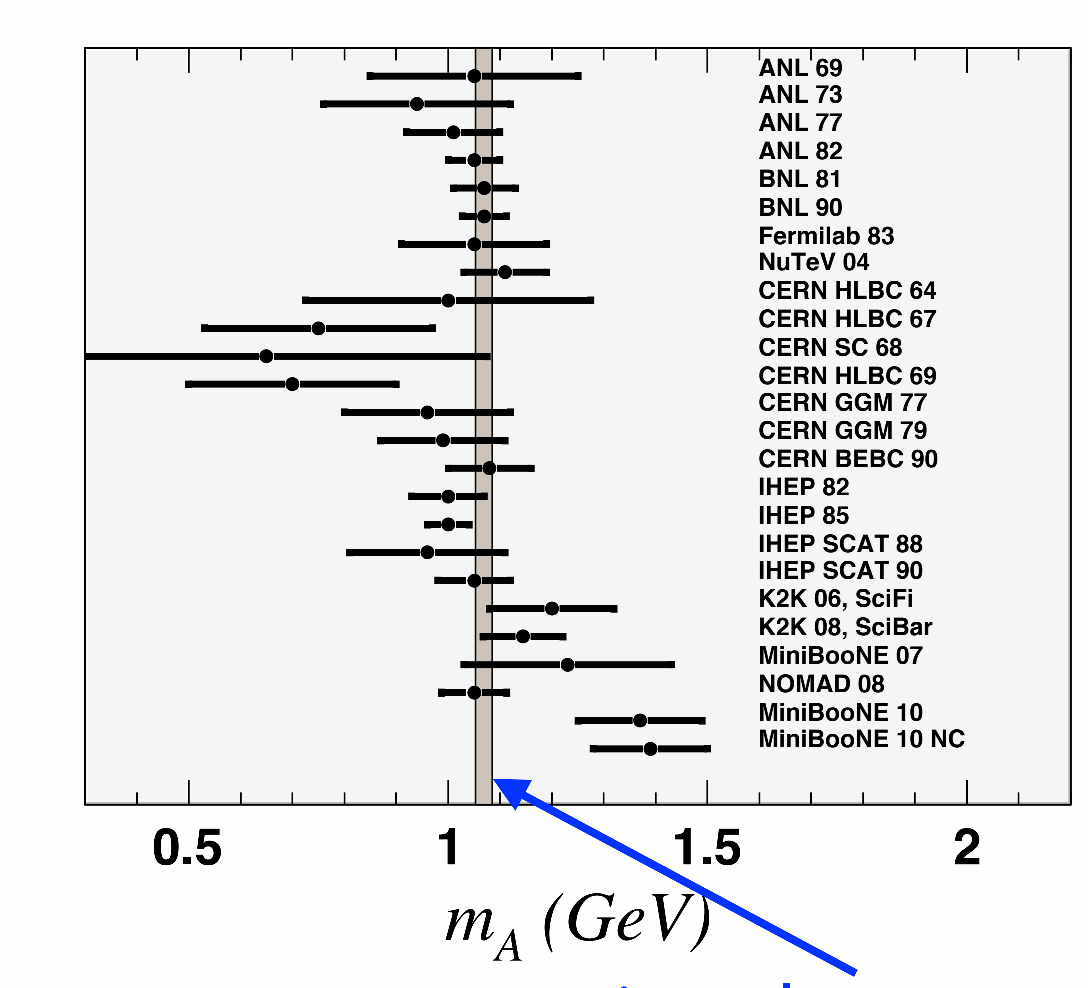
$$H_{\mu\nu} = \mathrm{Tr}[(\not{p}' + m_p)\Gamma_\mu(q)(\not{p} + m_n)\bar{\Gamma}_\nu(q)],$$

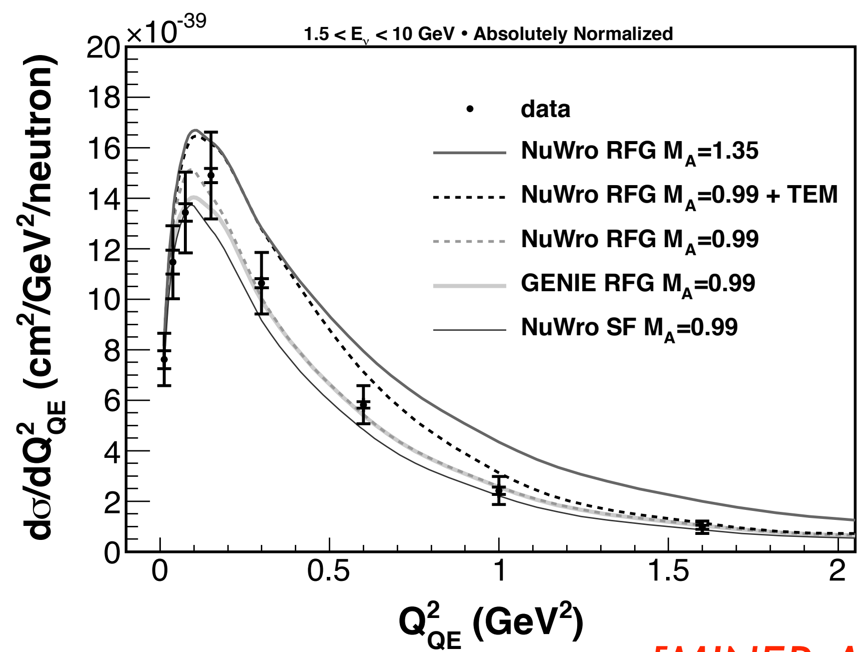
$$f(\boldsymbol{p}, q^0, \boldsymbol{q}) = \frac{m_T V}{4\pi^2} n_i(\boldsymbol{p})[1 - n_f(\boldsymbol{p} + \boldsymbol{q})] \frac{\delta(\epsilon_{\boldsymbol{p}} - \epsilon'_{\boldsymbol{p}+\boldsymbol{q}} + q^0)}{\epsilon_{\boldsymbol{p}} \epsilon'_{\boldsymbol{p}+\boldsymbol{q}}}$$

$$V = \frac{3\pi^2 A}{2p_F^3} \qquad n_i(\boldsymbol{p}) = \theta(p_F - |\boldsymbol{p}|), \quad n_f(\boldsymbol{p}') = \theta(p_F - |\boldsymbol{p}'|),$$

For the purposes of this talk, view nucleus as part of the detector (experiments are needed to calibrate)

Symptom of oversimplified form factor and nuclear models





[MINERvA, Phys.Rev.Lett. 111 (2013) 022502]

pion electroproduction *[Bernard et al 2002]*

- Discrepancies (~3 sigma) in CCQE measurements
- Unclear whether due to nuclear effects or nucleon-level amplitudes

Difficult measurements, but

your favorite signal

- (rhetorical) question: would we believe a collider measurement of [X] if it required a different value of m_W, or invoked a definition of m_W that could not be compared to other sources?

Math

Taylor expansions and fitting

Suppose we are given a set of data with errors and wish to determine derived quantities

In general, QM observable given by

$$\sigma_i \sim |A(x_i)|^2$$

Let us Taylor expand (e.g. σ=x.s., A=f.f., $x=q^2$)

$$A(x) = a_0 + a_1 x + a_2 x^2 + \ldots$$

Now consider (e.g. A real)

$$\chi^2 = \sum_i \frac{[\sigma_i - [A(x_i)]^2]^2}{d\sigma_i^2}$$ = fourth order polynomial in a_i

In finding minimum and $\Delta X^2 = 1$ intervals, etc., important in practice to know whether X^2 function is convex: is a local minimum necessarily a global minimum?

> "Virtually nothing is known about about finding global extrema in general..."
> Press et.al., Numerical Recipes,

Unfortunately, determining whether a general fourth-order polynomial is convex is NP hard

Fortunately, our fourth-order polynomial is special, and can be shown to obey "non-perverse convexity"

$$3[A(x_i)]^2 > \sigma_i \implies X^2 \text{ is convex}$$

i.e., unless errors are O(1), X^2 is convex and we may simply and efficiently "roll to the minimum"

proof (sketch):

$$M_I(a_i) = \text{convex} \quad \Rightarrow \quad \sum_I M_I(a_i) = \text{convex}$$

$$M(a_i) = \text{convex} \quad \Leftrightarrow \quad \frac{\partial^2 M}{\partial a_i \partial a_j} = \text{positive definite}$$

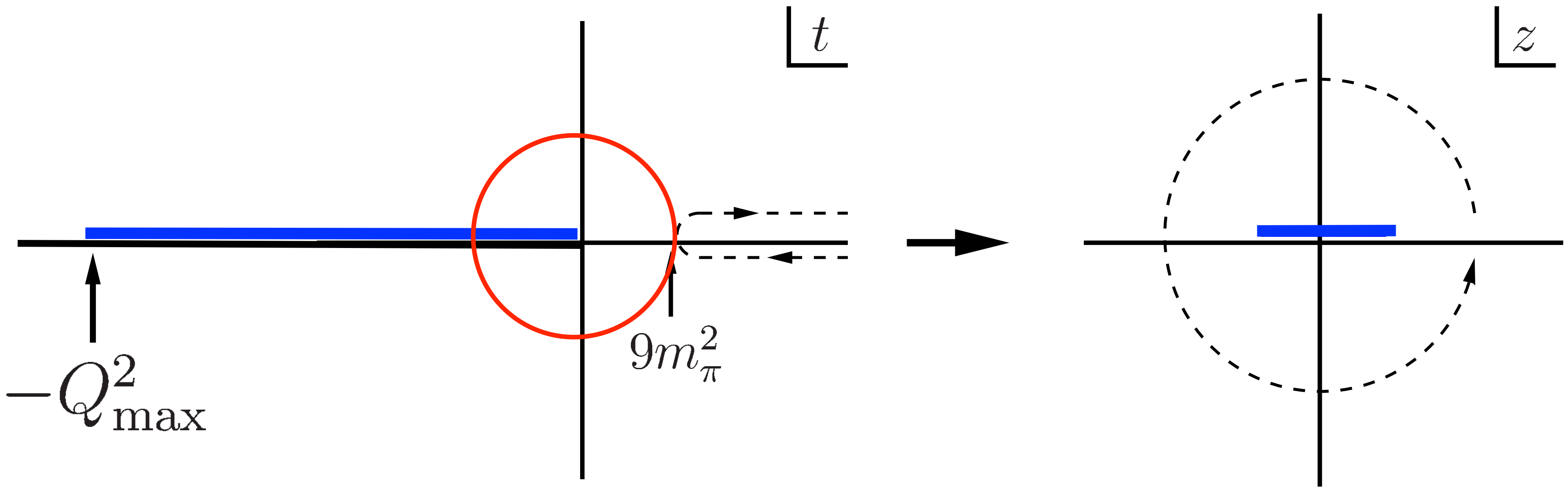
$$M(a_i) = (\sigma - [\sum_i a_i x^i]^2)^2$$

$$\Rightarrow \frac{\partial^2 M}{\partial a_i \partial a_j} = 4x^{i+j}\left[3\left(\sum_k a_k x^k\right)^2 - \sigma\right]$$

$$\frac{\partial^2 M}{\partial a_i \partial a_j} \propto \begin{pmatrix} 1 & x & x^2 & \cdots \\ x & x^2 & x^3 & \cdots \\ x^2 & x^3 & x^4 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} 1 \\ x \\ x^2 \\ \vdots \end{pmatrix} (1 \quad x \quad x^2 \cdots) = LL^T$$

$$\lambda = v^T \frac{\partial^2 M}{\partial a_i \partial a_j} v = (L^T v)^2 \geq 0$$

Unfortunately, a simple Taylor expansion of hadronic amplitudes has finite (small) radius of convergence



Fortunately, the analytic structure of amplitudes allows us to "resum" by change of variables into expansion covering the entire physical region

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

$9m_\pi^2$ (isoscalar channel)

point mapping to z=0
(scheme choice)

Particle (nucleon) physics

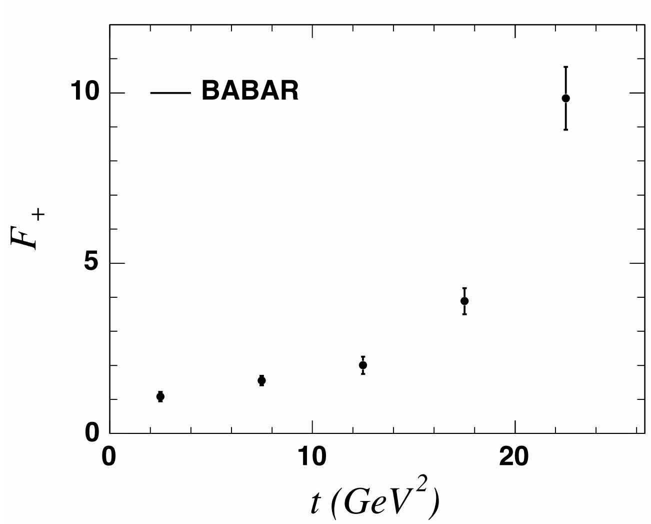
- basic idea: small expansion parameter, z, with *order unity* expansion coefficients

$$F(q^2) = \sum_{n=0}^{\infty} a_n z(q^2)^n$$

- in fact, a little better, e.g.

$$\sum_{n=0}^{\infty} a_n^2 < \infty \quad \Rightarrow$$ a_n *smaller* for large n

The z expansion has become a standard tool for meson transitions (e.g. |Vub| determinations in B → π l ν)



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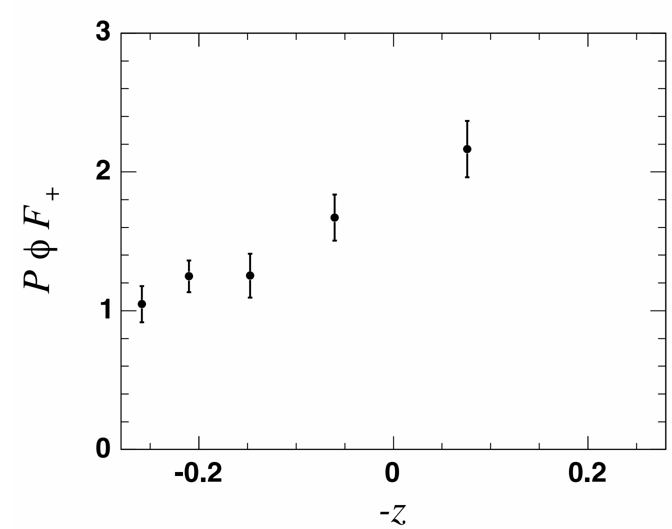


[Bourrely et al 1981]

[Boyd, Grinstein, Lebed 1995]

[Lellouch et al 1996]

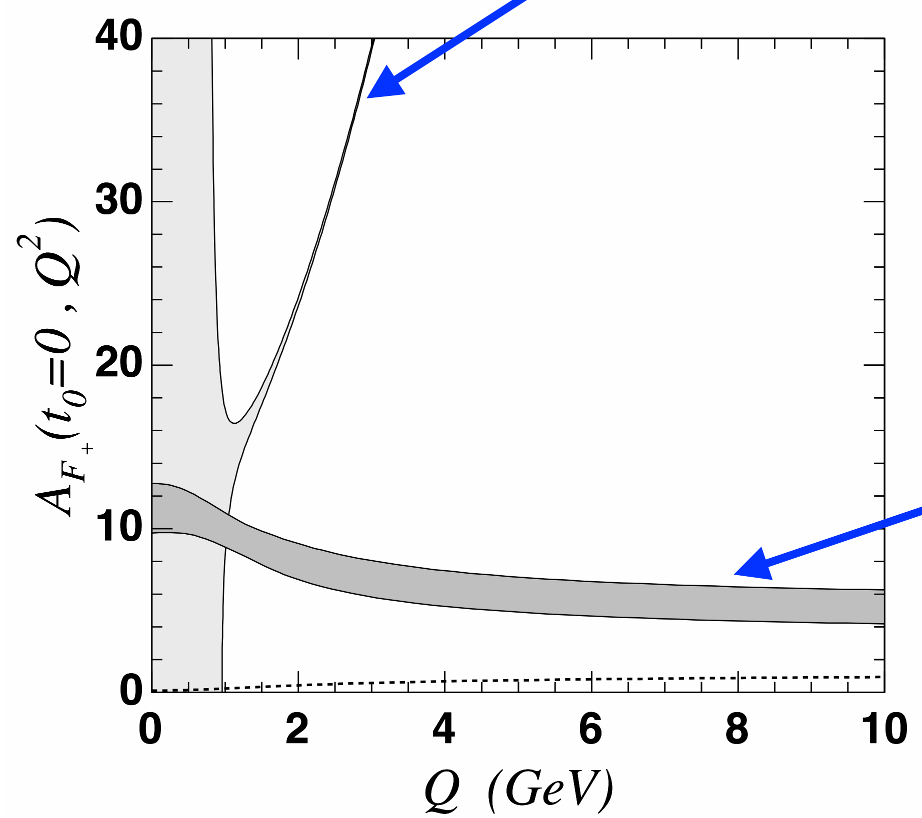
[Arnesen et al 2005]

[Becher, Hill 2006] ....

Note that the real power of the expansion is based on observation of O(1) coefficients, not unitarity bounds. E.g., for K→π vector form factor, can *measure* bound:

unitarity bound on A (require exclusive rate< inclusive rate)

$$A = \sqrt{\sum_k \frac{a_k^2}{a_0^2}}$$



actual size of A (measured τ→Kπν)

[RJH Phys.Rev. D74 (2006) 096006]

scheme choice to evaluate OPE for inclusive rate

⇒ Unitarity bound either uncertain (low Q) or overestimates bound (high Q)

For nucleon form factors, unitarity even less relevant, as dominant dispersive contribution to form factors is from states below NN threshold

At least until very recently, curvature never measured in any meson transition form factor

Process	CKM element	$\lvert z\rvert_{\max}$
$\pi^+ \to \pi^0$	V_{ud}	3.5×10^{-5}
$B \to D$	V_{cb}	0.032
$K \to \pi$	V_{us}	0.047
$D \to K$	V_{cs}	0.051
$D \to \pi$	V_{cd}	0.17
$B \to \pi$	V_{ub}	0.28

Process	a_1/a_0	Reference
$B \to D$	-2.6 ± 2.3	[13]
$K^+ \to \pi^0$	-0.2 ± 0.2	[14]
$K_L \to \pi^\pm$	-0.5 ± 0.2	[15]
	0.0 ± 0.3	[16]
	-0.2 ± 0.2	[17]
$D \to K$	$-2.7 \pm 0.5 \pm 0.4$	[18]
	$-2.2 \pm 0.4 \pm 0.4$	[19]
	$-3.2 \pm 0.5 \pm 0.2$	[20]
$D \to \pi$	$-2.3 \pm 0.7 \pm 1.3$	[18]
	$-1.6 \pm 0.5 \pm 1.0$	[20]
$B \to \pi$	$-1.3 \pm 0.6 \pm 2.3$	[21]
	$-1.9 \pm 0.3 \pm 1.1$	[12]
	$-1.3 \pm 0.8 \pm 2.2$	[22]

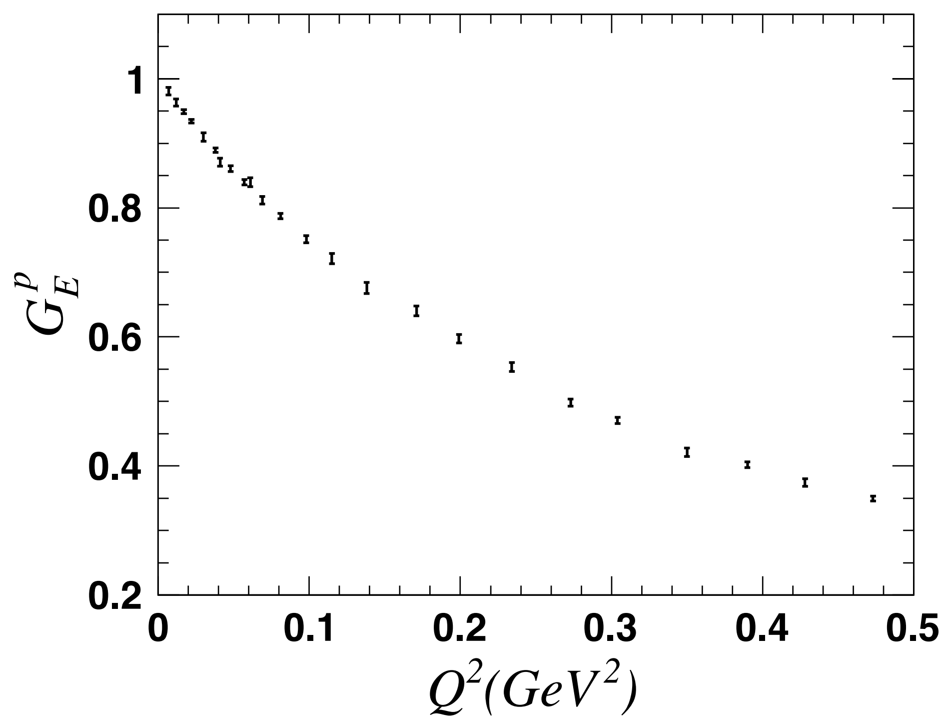
[RJH, eConf C060409 (2006) 027]

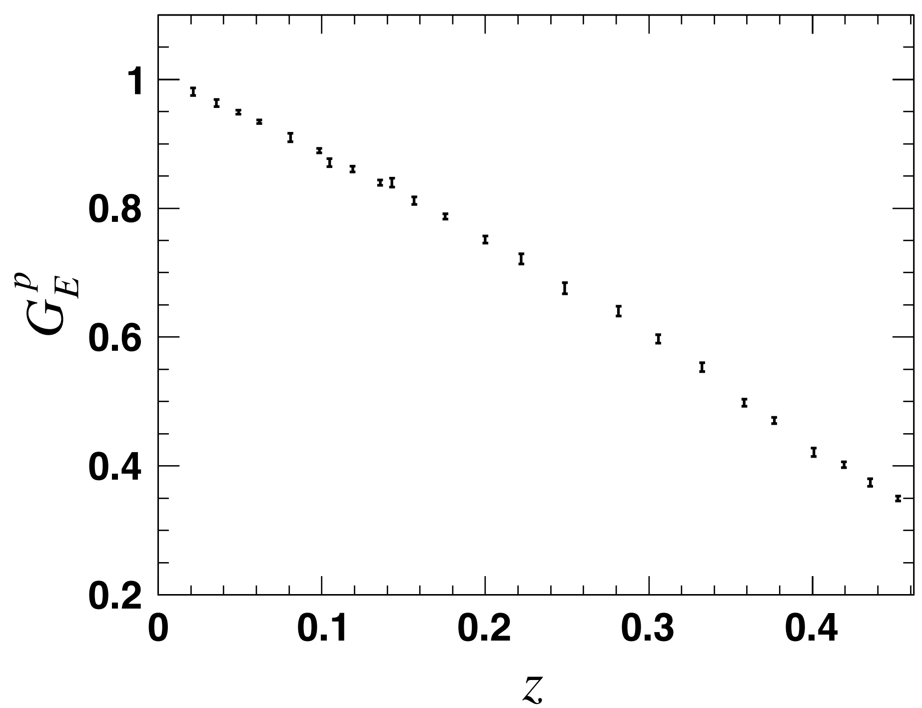
First hints perhaps seen in $B \to \pi$ (to be expected, cf. above)

[BaBar, Phys.Rev. D83 (2011) 052011]

Bringing the z expansion into the domain of baryon form factors

- study of vector dominance models, ππ approximation to isovector form factors: expect O(1) is really order 1 (e.g. not 10)





- more concretely, fits to data yield

[Bhattacharya, Hill, Paz, Phys.Rev. D84 (2011) 073006]

$$a_0 \equiv 1\,, \quad a_1 = -1.01(6)\,, \quad a_2 = -1.4^{+1.1}_{-0.7}\,, \quad a_3 = 2^{+2}_{-6}$$

- to assign error, constrain coefficients, e.g. <5 (conservative) or <10 (very conservative)

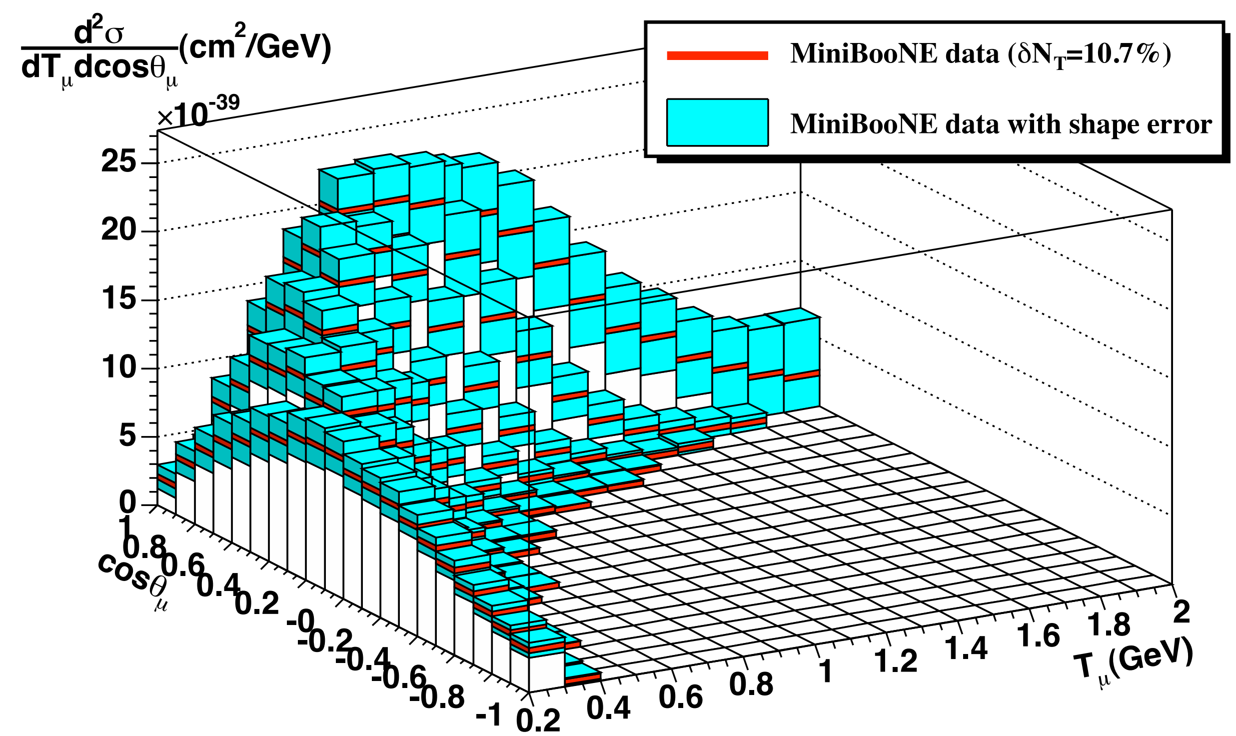
- as for mesons, also for nucleons: curvature as-yet unmeasured (so in practice, shape is determined by one number)

Parameter	Value
$\lvert V_{ud}\rvert$	0.9742
μ_p	2.793
μ_n	-1.913
m_μ	0.1057 GeV
G_F	1.166×10^{-5} GeV^{-2}
m_N	0.9389 GeV
$F_A(0)$	-1.269
ϵ_b	0.025 GeV
p_F	0.220 GeV

$$E_{ij} = (\delta\sigma_i)^2\delta_{ij} + (\delta N)^2\sigma_i\sigma_j$$

$$\chi^2 = \sum_{ij}(\sigma_i^{\text{expt.}} - \sigma_i^{\text{theory}})E_{ij}^{-1}(\sigma_j^{\text{expt.}} - \sigma_j^{\text{theory}})$$

Fit to double differential CCQE data from MiniBooNE

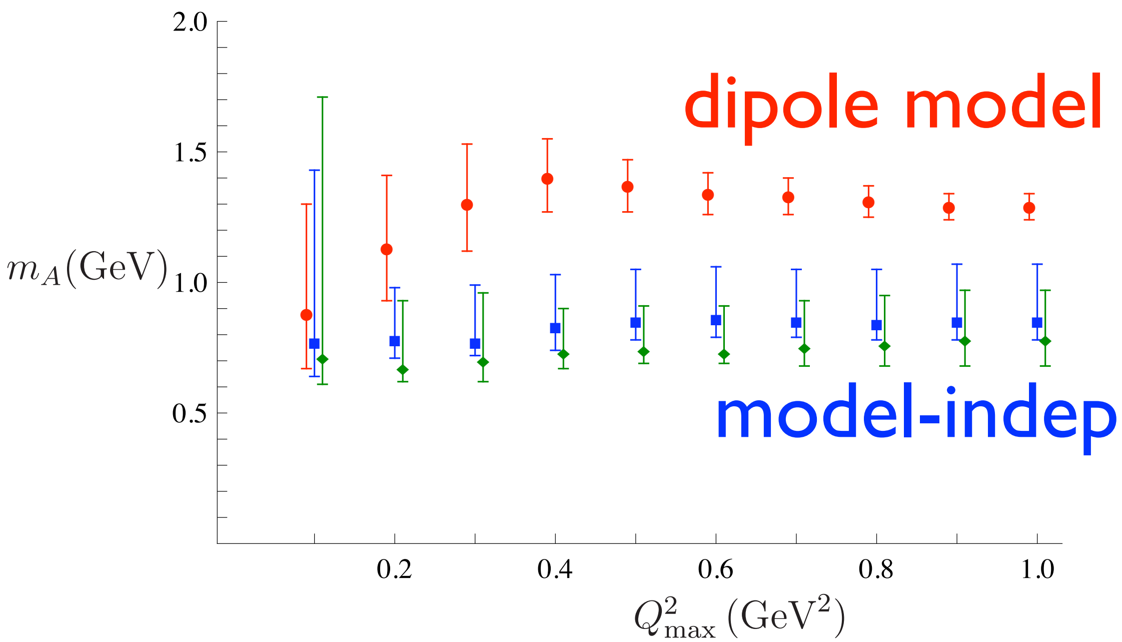


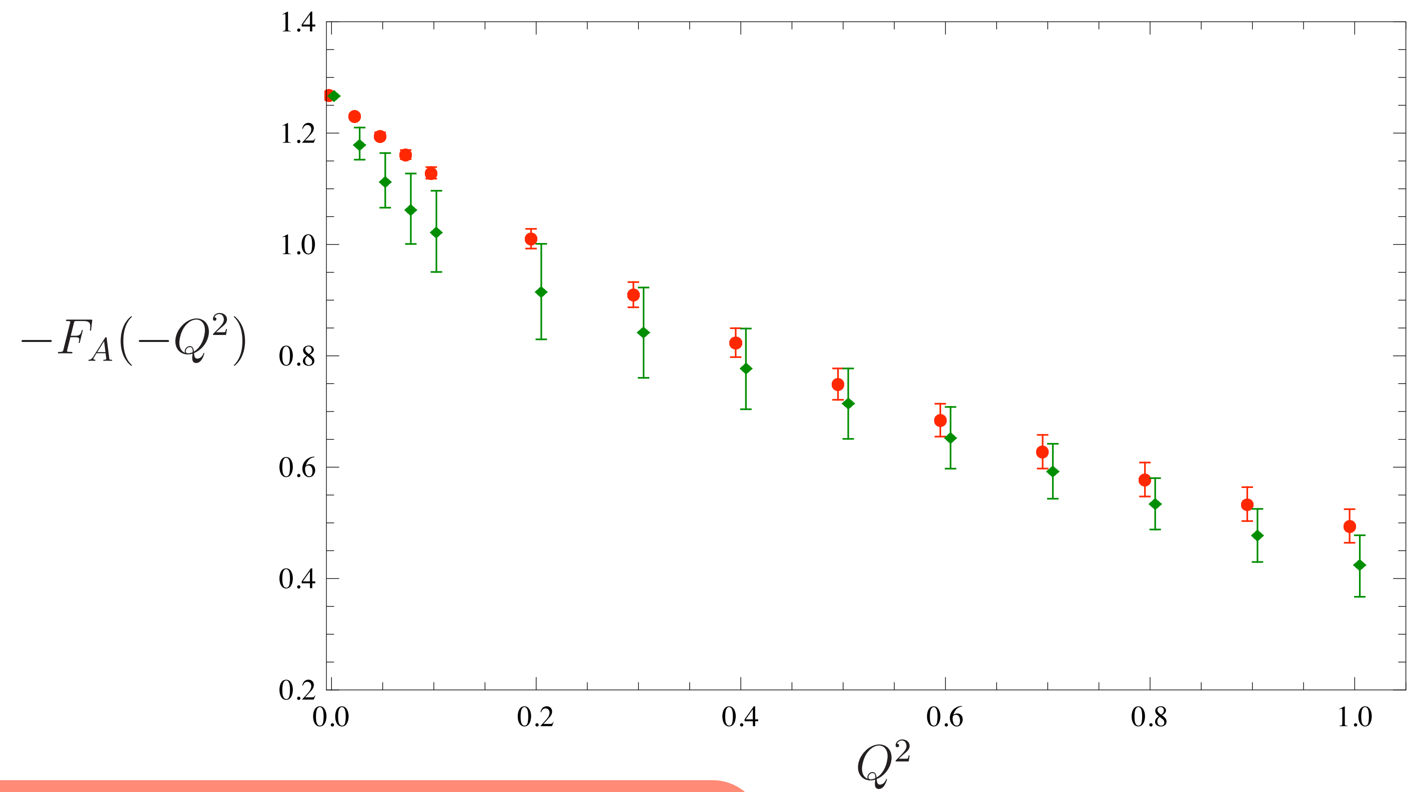
[MiniBooNE, PRD81, 092005 (2010)]

Assume Relativistic Fermi Gas nuclear model *[Smith and Moniz (1972)]*

Results for axial form factor:

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2}q^2 + \dots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$





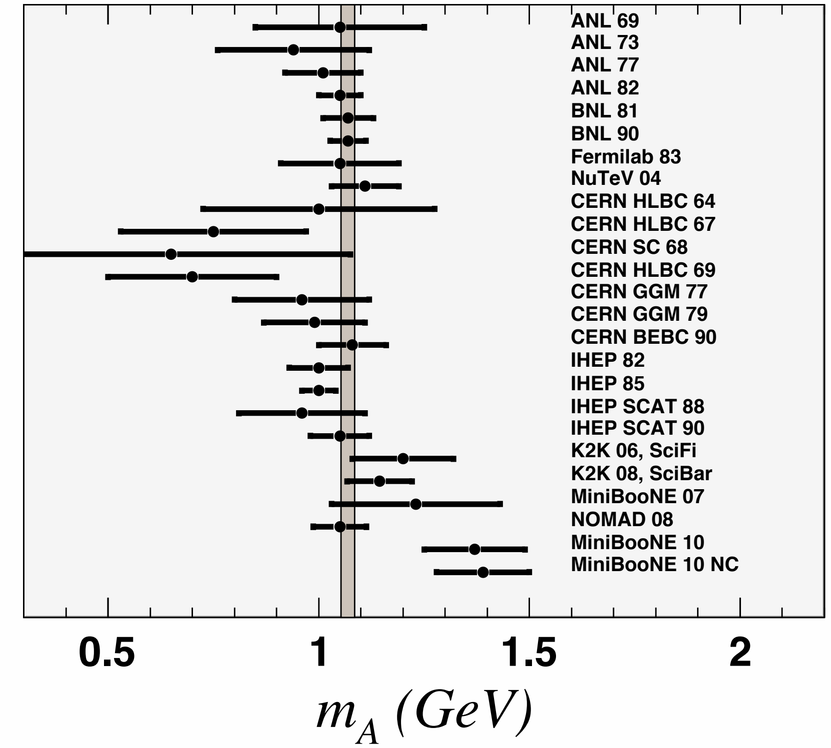
$$\boxed{m_A = 0.85^{+0.22}_{-0.07} \pm 0.09\,\mathrm{GeV} \qquad \text{(neutrino scattering)}}$$

$$m_A^{\mathrm{dipole}} = 1.29 \pm 0.05\,\mathrm{GeV}$$

$$a_0 = F_A(0) = -1.269, \quad a_1 = 2.9^{+1.1}_{-1.0}, \quad a_2 = -8^{+6}_{-3}$$

- again, no measurable curvature (in z)

Revisit pion electroproduction

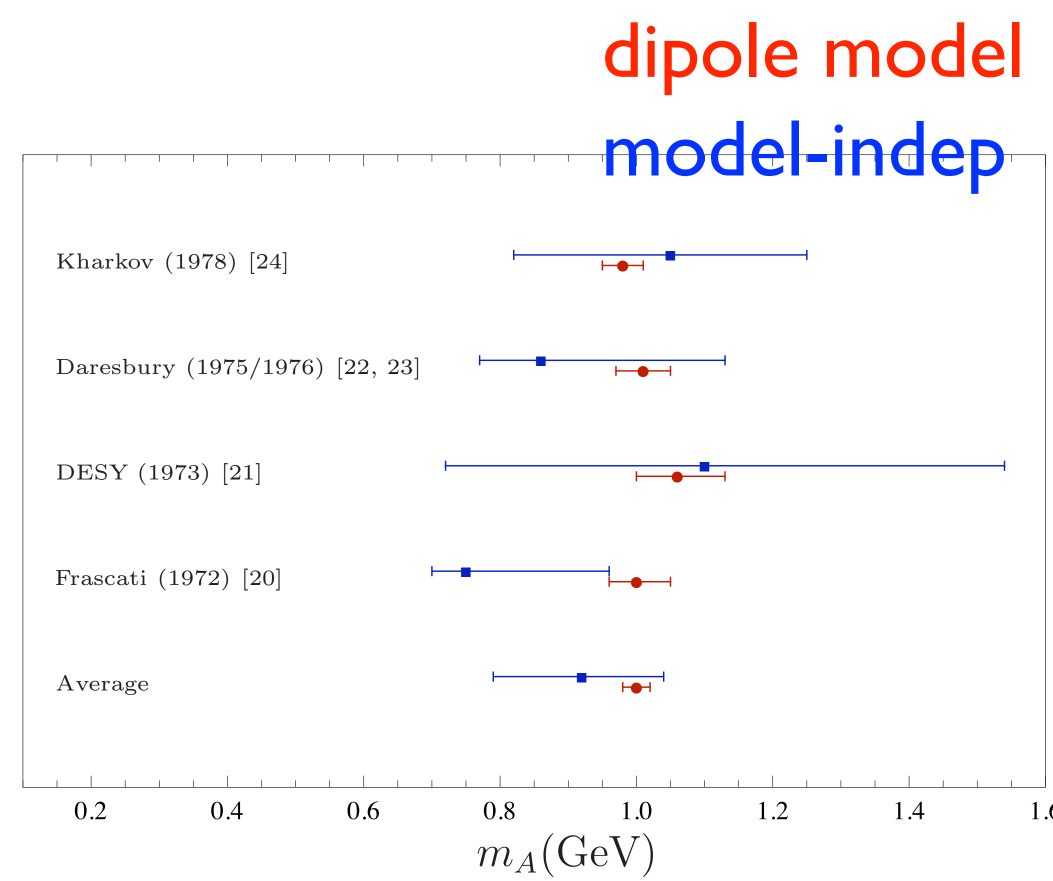


Experimental anomalies are between

a) high and low energy neutrino data

b) neutrino data and electroproduction data

$$m_A = 0.92^{+0.12}_{-0.13} \pm 0.08\,\mathrm{GeV} \qquad \text{(electroproduction)}$$



- World average strongly affected by dipole assumption
- Extrapolation beyond chiral regime
- Naive/absent treatment of radiative corrections

Summary

- degeneracy between flux, nucleon-level, and nuclear uncertainties in neutrino-nucleus scattering and related observables
- caution warranted. cf. “proton radius puzzle” in e.m. Here radius determined to ~2% by e-p scattering. Similar uncertainty often claimed for axial radius
- z expansion applied to nucleon f.f.s (implemented in GENIE: A. Meyer)
- lattice poised to make critical contribution at nucleon level, breaking the above degeneracy
- experimental input important to constrain nuclear models